

SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours

Maximum Marks : 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	2(2)	–	1(3)	–	3(5)
2.	Inverse Trigonometric Functions	1(1)	1(2)	–	–	2(3)
3.	Matrices	2(2)	1(2)	–	–	3(4)
4.	Determinants	1(1)*	–	–	1(5)*	2(6)
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	3(8)
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	3(9)
7.	Integrals	1(1)*	1(2)*	1(3)*	–	3(6)
8.	Application of Integrals	1(1)	1(2)	1(3)	–	3(6)
9.	Differential Equations	1(1)*	1(2)*	1(3)	–	3(6)
10.	Vector Algebra	3(3)#	1(2)*	–	–	4(5)
11.	Three Dimensional Geometry	4(4)	–	–	1(5)*	5(9)
12.	Linear Programming	–	–	–	1(5)*	1(5)
13.	Probability	1(4)	2(4)	–	–	3(8)
	Total	18(24)	10(20)	7(21)	3(15)	38(80)

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

MATHEMATICS

Time allowed : 3 hours

Maximum marks : 80

General Instructions :

1. This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.
2. Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.
3. Both Part-A and Part-B have internal choices.

Part -A :

1. It consists of two Sections-I and II.
2. Section-I comprises of 16 very short answer type questions.
3. Section-II contains 2 case study-based questions.

Part - B :

1. It consists of three Sections-III, IV and V.
2. Section-III comprises of 10 questions of 2 marks each.
3. Section-IV comprises of 7 questions of 3 marks each.
4. Section-V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A**Section - I**

1. Write the cofactor of the element a_{31} in $A = \begin{pmatrix} 3 & 2 & 6 \\ 5 & 0 & 7 \\ 3 & 8 & 5 \end{pmatrix}$.

ORIf A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

2. Evaluate : $\tan^{-1} \left[2 \cos \left(2 \sin^{-1} \frac{1}{2} \right) \right]$

3. Find the integrating factor of the differential equation $\left\{ \frac{e^{-2\sqrt{x}}}{\sqrt{x}} + \frac{y}{\sqrt{x}} \right\} \frac{dx}{dy} = 1 (x \neq 0)$.

ORFind order and degree of the equation $\left(\frac{d^3 y}{dx^3} \right)^4 + \left(\frac{d^2 y}{dx^2} \right)^3 + \frac{dy}{dx} + 4y = \sin x$.

4. If $f(x) = x^2 - 4x + 1$, find $f(A)$, where $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.
5. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

OR

Find the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on $2\hat{i} + 6\hat{j} + 3\hat{k}$.

6. Check whether the function $f(x) = x^3 - 3x^2 - x$ is one-one or not?
7. Evaluate : $\int_1^3 (x-1)(x-2)(x-3)dx$

OR

Evaluate : $\int_{-\pi}^{\pi} x^{10} \sin^7 x \, dx$

8. Check whether the lines having direction ratios $(\sqrt{3}-1, -\sqrt{3}-1, 4)$ and $(-\sqrt{3}+1, \sqrt{3}+1, -4)$ are perpendicular to each other.
9. If the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 4x\hat{j} + y\hat{k}$ are parallel, then the values of x and y .

OR

Find the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, 2, 3)$ in the ratio $2 : 3$ externally.

10. Find vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$.
11. Let $A = \{1, 2, 3, 4\}$. Show that $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ is a bijection from A to A ?
12. If \vec{a} and \vec{b} are unit vectors enclosing an angle θ and $|\vec{a} + \vec{b}| < 1$, find the value of θ .
13. Find the area of the region bounded between the line $x = 2$ and the parabola $y^2 = 8x$.
14. Find equation of a line which passes through the point $(1, 2, 3)$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.
15. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$, such that $A + B + C$ is a zero matrix, then find the matrix C .

16. If the line joining $(2, 3, -1)$ and $(3, 5, -3)$ is perpendicular to the line joining $(1, 2, 3)$ and $(3, 5, \lambda)$, then find the value of λ .

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A teacher arranged a surprise game for students of a classroom having 5 students, namely Amit, Aruna, Eklavya, Yash and Samina. He took a bag containing tickets numbered 1 to 11 and told each student to draw two tickets without replacement.

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- (i) Probability that both tickets drawn by Amit shows even number, is
 (a) $1/11$ (b) $2/11$ (c) $3/11$ (d) $4/11$
- (ii) Probability that both tickets drawn by Aruna shows odd number, is
 (a) $1/11$ (b) $2/11$ (c) $3/11$ (d) $4/11$
- (iii) When tickets are drawn by Eklavya, find the probability that number on one ticket is a multiple of 4 and on other ticket is a multiple of 5.
 (a) $4/55$ (b) $6/55$ (c) $7/55$ (d) None of these
- (iv) When tickets are drawn by Yash, find the probability that number on one ticket is a prime number and on other ticket is a multiple of 4.
 (a) $3/11$ (b) $5/11$ (c) $6/11$ (d) $2/11$
- (v) When tickets are drawn by Samina, find the probability that first ticket drawn shows an even number and second ticket drawn shows an odd number.
 (a) $2/11$ (b) $3/11$ (c) $5/11$ (d) $8/11$

18. An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 4000 l of water, that comes out from a tube well.



Based on above information, answer the following questions.

- (i) If the length, width and height of the open tank be x , x and y m respectively, then surface area of tank is given by
 (a) $S = x^2 + 2xy$ (b) $S = 2x^2 + 4xy$ (c) $S = 2x^2 + 2xy$ (d) $S = 2x^2 + 8xy$
- (ii) The relation between x and y is
 (a) $x^2y = 4$ (b) $xy^2 = 4$ (c) $x^2y^2 = 4$ (d) $xy = 4$
- (iii) The outer surface area of tank will be minimum when depth of tank is equal to
 (a) half of its width (b) its width (c) $\left(\frac{1}{4}\right)^{\text{th}}$ of its width (d) $\left(\frac{1}{3}\right)^{\text{rd}}$ of its width
- (iv) The cost of material will be least when width of tank is equal to
 (a) half of its depth (b) twice of its depth (c) $\left(\frac{1}{4}\right)^{\text{th}}$ of its depth (d) thrice of its depth

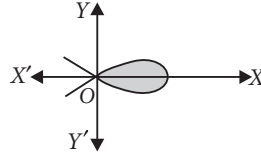


- (v) If cost of aluminium sheet is ₹ 360/m², then the minimum cost for the construction of tank will be
 (a) ₹ 2320 (b) ₹ 3320 (c) ₹ 4320 (d) ₹ 5320

PART - B

Section - III

19. Find the equations of the tangent and the normal to the curve $y = x^3$ at the point $P(1, 1)$.
20. Express $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$, $x \in \left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$ in the simplest form.
21. Find the area of region bounded by the curve $y^2 = x(1-x)^2$, shown in following figure.



22. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. If an orator is chosen at random, find the probability that a male person is selected. Assume that there are equal number of men and women.

23. Evaluate : $\int \frac{1}{\sqrt{1-\sin x}} dx$

OR

Evaluate : $\int \frac{dx}{1-2\sin x \cos x}$

24. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = AA^T$, then find the value of $5a + b$.

25. Find the projection of the vector $2\hat{i} - 3\hat{j} - 6\hat{k}$ on vector joining the points $(5, 6, -3)$ and $(3, 4, -2)$.

OR

If $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{k}$, find $|\vec{b} \times 2\vec{a}|$.

26. Suppose that two cards are drawn at random from a deck of 52 cards. Let X be the number of aces obtained. Then, find the probability distribution of X .

27. If $y = \sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

28. Find the solution of the differential equation $\frac{dy}{dx} = \frac{y(1+x)}{x(y-1)}$.

OR

Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$; $y = 0, x = 0$.

Section - IV

29. Show that the curve for which the normal at every point passes through a fixed point is a circle.
30. Find the point on the parabola $y^2 = 2x$ which is closed to the point $(1, 4)$.
31. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then find $\frac{dy}{dx}$.

OR

Differentiate $\frac{2^{x^2+x+1} \cdot \sqrt{4x-1}}{(x^2-1)^{3/2}}$ w.r.t. x .

32. Let $f: A \rightarrow B$ be a function defined as $f(x) = \frac{2x+3}{x-3}$, where $A = R - \{3\}$ and $B = R - \{2\}$. Is the function f one-one and onto?

33. Find the area of the region bounded by the curve $y = x^2 + x$, the x -axis and the lines $x = 2, x = 5$.

34. Evaluate: $\int_0^1 \tan^{-1} x \, dx$

OR

Evaluate: $\int_1^2 \frac{dx}{x(1+x^2)}$

35. If $f(x)$ is continuous at $x = 0$, where

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{for } x > 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & \text{for } x < 0 \end{cases}, \text{ then find } f(0).$$

Section-V

36. Find the vector and cartesian equation of the line through the point $\hat{i} + \hat{j} - 3\hat{k}$ and perpendicular to the lines $\vec{r} = 2\hat{i} - 3\hat{j} + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ and $\vec{r} = 3\hat{i} - 5\hat{j} + \mu(\hat{i} + \hat{j} + \hat{k})$.

OR

The four points $A(3, 2, -5), B(-1, 4, -3), C(-3, 8, -5)$ and $D(-3, 2, 1)$ are coplanar. Find the equation of the plane containing them.

37. Find the minimum value of $Z = 3x + 4y + 270$ subject to the constraints

$$\begin{aligned} x + y &\leq 60 \\ x + y &\geq 30 \\ x &\leq 40, y \leq 40 \\ x &\geq 0, y \geq 0 \end{aligned}$$

OR

Find the point for which the maximum value of $Z = x + y$ subject to the constraints $2x + 5y \leq 100$,

$$\frac{x}{25} + \frac{y}{50} \leq 1, x \geq 0, y \geq 0 \text{ is obtained.}$$

38. If $A = \begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$, find A^{-1} . Using A^{-1} solve the following system of equations :

$$\begin{aligned} 2x + 3y + 7z &= 12 \\ 3x - 2y - z &= 0 \\ x + y + 2z &= 4 \end{aligned}$$

OR

$$\text{If } A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}, \text{ find } (\text{adj } A^{-1}).$$



1. We have, $A = \begin{pmatrix} 3 & 2 & 6 \\ 5 & 0 & 7 \\ 3 & 8 & 5 \end{pmatrix}$

Cofactor of $a_{31} = C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 6 \\ 0 & 7 \end{vmatrix} = 14$

OR

Given, A is a square matrix of order 3
 $\therefore |2A| = 2^3|A| = 8|A| = k|A|$ (Given)
 $\Rightarrow k = 8$

2. We have, $\tan^{-1} \left\{ 2 \cos \left(2 \sin^{-1} \left(\frac{1}{2} \right) \right) \right\}$
 $= \tan^{-1} \left\{ 2 \cos \left(2 \times \frac{\pi}{6} \right) \right\} \quad \left[\because \sin^{-1} \frac{1}{2} = \frac{\pi}{6} \right]$
 $= \tan^{-1} \left\{ 2 \cos \frac{\pi}{3} \right\} = \tan^{-1} \left[2 \times \frac{1}{2} \right] = \tan^{-1} 1 = \frac{\pi}{4}$

3. We have, $\frac{dy}{dx} - \frac{1}{\sqrt{x}} \cdot y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

This is of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{-1}{\sqrt{x}}$
 and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

I.F. = $e^{\int P dx} = e^{\int \frac{-1}{\sqrt{x}} dx} = e^{-2\sqrt{x}}$.

OR

Highest order derivative is $\left(\frac{d^3 y}{dx^3} \right)$. So, its order is 3
 and degree is 4.

4. We have, $f(x) = x^2 - 4x + 1 \Rightarrow f(A) = A^2 - 4A + I$

$\therefore A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \therefore A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$\therefore f(A) = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 $= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. The unit vector in the direction of a vector \vec{a} is given

by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$. Now, $|\vec{a}| = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$

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Therefore, $\hat{a} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{29}} = \frac{2}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$

OR

Let $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

Then, $\vec{a} \cdot \vec{b} = (7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})$
 $= 14 + 6 - 12 = 8$

Also, $|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = 7$

\therefore Projection of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$

6. We have $f(x) = x^3 - 3x^2 - x$

Clearly, $f(1) = 1 - 3 - 1 = -3$

and $f(-1) = -1 - 3 + 1 = -3$

\Rightarrow Distinct elements have same image, therefore f is not one-one.

7. Let $I = \int_1^3 (x-1)(x-2)(x-3) dx$

$= \int_1^3 (x^3 - 6x^2 + 11x - 6) dx = \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{11x^2}{2} - 6x \right]_1^3$

$= \left[\frac{81}{4} - \frac{162}{3} + \frac{99}{2} - 18 - \left(\frac{1}{4} - \frac{6}{3} + \frac{11}{2} - 6 \right) \right] = 0$

OR

Let $I = \int_{-\pi}^{\pi} x^{10} \sin^7 x dx$

Also, let $f(x) = x^{10} \sin^7 x$

Then, $f(-x) = (-x)^{10} [\sin(-x)]^7 = -x^{10} \sin^7 x = -f(x)$

$\Rightarrow f(x)$ is an odd function.

$\therefore I = \int_{-\pi}^{\pi} x^{10} \sin^7 x dx = 0$

8. Here, $a_1 = \sqrt{3} + 1$, $b_1 = -\sqrt{3} + 1$, $c_1 = 4$ and

$a_2 = -\sqrt{3} + 1$, $b_2 = \sqrt{3} + 1$, $c_2 = -4$

Since, $\frac{a_1}{a_2} = -1$, $\frac{b_1}{b_2} = -1$ and $\frac{c_1}{c_2} = -1$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

\Rightarrow Direction ratios of lines are proportional.

Hence, the lines are parallel to each other.

9. Let $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 4x\hat{j} + y\hat{k}$

Since, \vec{a} and \vec{b} are parallel

$\therefore \vec{a} = m\vec{b}$, for some $m \in R$

$\Rightarrow 3\hat{i} + 2\hat{j} - \hat{k} = m(6\hat{i} - 4x\hat{j} + y\hat{k})$

$\Rightarrow 3 = 6m \Rightarrow m = \frac{1}{2}$

Also, $-4xm = 2 \Rightarrow \frac{-4x}{2} = 2 \Rightarrow x = -1$

and $ym = -1 \Rightarrow \frac{y}{2} = -1 \Rightarrow y = -2$

OR

Let $C(x, y, z)$ divides the line segment joining the points $A(-2, 3, 5)$ and $B(1, 2, 3)$ in the ratio $2 : 3$ externally.

Now, $\vec{c} = \frac{2\vec{b} - 3\vec{a}}{2 - 3}$, where \vec{a} , \vec{b} and \vec{c} are position vectors of A , B and C respectively.

$= -1[2(\hat{i} + 2\hat{j} + 3\hat{k}) - 3(-2\hat{i} + 3\hat{j} + 5\hat{k})]$

$= -1(8\hat{i} - 5\hat{j} - 9\hat{k}) = -8\hat{i} + 5\hat{j} + 9\hat{k}$

So, co-ordinates of $C \equiv (-8, 5, 9)$

10. Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then,

$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$

Hence, the required equation of the plane is

$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k} \right) = \frac{6}{\sqrt{29}}$

11. Here $f(1) = 2, f(2) = 3, f(3) = 4, f(4) = 1$

Since no two elements have the same image. So f is one-one. Also, every elements has atleast one pre-image. So, f is onto.

Thus f is bijective.

12. $|\vec{a} + \vec{b}| < 1 \Rightarrow |\vec{a} + \vec{b}|^2 < 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} < 1 \Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} < 1$

$\Rightarrow \vec{a} \cdot \vec{b} < -\frac{1}{2} \Rightarrow |\vec{a}| |\vec{b}| \cos \theta < -\frac{1}{2}$

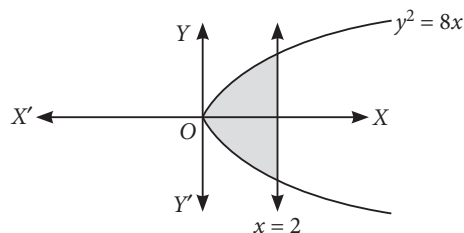
$\Rightarrow 1 \times 1 \times \cos \theta < -\frac{1}{2} \Rightarrow \cos \theta < -\frac{1}{2}$

$\Rightarrow -1 \leq \cos \theta < -\frac{1}{2} \Rightarrow \pi \geq \theta > \frac{2\pi}{3}$

13. We have, $y^2 = 8x$ and $x = 2$

\therefore Required area = Area of shaded region

$= 2 \cdot \int_0^2 \sqrt{8x} dx = 4\sqrt{2} \left[\frac{2}{3} x^{3/2} \right]_0^2 = \frac{32}{3}$ sq. units.



14. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$.

We know that the line which passes through point \vec{a} and parallel to \vec{b} is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a constant.

$\therefore \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$ is the required equation of the line.

15. We have, $A + B + C = O \Rightarrow C = -[A + B]$

$\Rightarrow C = (-1) \left\{ \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix} \right\} = (-1) \begin{bmatrix} 8 & 4 \\ -2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$

16. D.R.'s of the two lines are $1, 2, -2$ and $2, 3, \lambda - 3$.

Since, lines are perpendicular

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 1 \times 2 + 2 \times 3 - 2(\lambda - 3) = 0 \Rightarrow \lambda = 7$

17. (i) (b) : Total number of tickets = 11

Let the event A = First ticket shows even number

and B = Second ticket shows even number

Now, $P(\text{Both ticket shows even number}) = P(A) \cdot P(B|A)$

$= \frac{5}{11} \cdot \frac{4}{10} = \frac{2}{11}$

(ii) (c) : Let the event A = First ticket shows odd number and B = Second ticket shows odd number

$P(\text{Both ticket shows odd number})$

$= \frac{6}{11} \times \frac{5}{10} = \frac{3}{11}$

(iii) (a) : Required probability = $P(\text{one number is a multiple 4 and other is a multiple 5})$

$= P(\text{multiple of 5 on first ticket and multiple of 4 on second ticket}) + P(\text{multiple of 4 on first ticket and multiple of 5 on second ticket})$

$= \frac{2}{11} \cdot \frac{2}{10} + \frac{2}{11} \times \frac{2}{10}$

$= \frac{4}{110} + \frac{4}{110} = \frac{8}{110} = \frac{4}{55}$

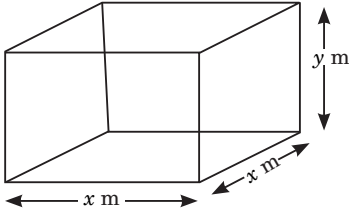
(iv) (d) : Required probability = $P(\text{one ticket with prime number and other ticket with a multiple of 4})$

$= \frac{5}{11} \times \frac{2}{10} + \frac{2}{11} \times \frac{5}{10} = \frac{10}{110} + \frac{10}{110} = \frac{2}{11}$

(v) (b) : Let the event A = First ticket shows even number and B = Second ticket shows odd number
 Now, $P(\text{First ticket shows an even number and second ticket shows an odd number}) = P(A) \cdot P(B|A)$

$$= \frac{5}{11} \times \frac{6}{10} = \frac{30}{110} = \frac{3}{11}$$

18. (i) (d) : Since the tank is open from the top, therefore the total surface area is
 $= 2(x \times x + 2(xy + yx)) = 2(x^2 + 2(2xy)) = 2x^2 + 8xy$



(ii) (a) : Since, volume of tank should be 4000 l.
 $\therefore x^2 y \text{ m}^3 = 4000 \text{ l} = 4 \text{ m}^3$ [$\because 1 \text{ litre} = 0.001 \text{ m}^3$]
 So, $x^2 y = 4$

(iii) (a) : Let S be the outer surface area of tank.
 Then, $S = x^2 + 4xy$
 $\Rightarrow S(x) = x^2 + 4x \cdot \frac{4}{x^2} = x^2 + \frac{16}{x}$ [$\because x^2 y = 4$]
 $\Rightarrow \frac{dS}{dx} = 2x - \frac{16}{x^2}$ and $\frac{d^2S}{dx^2} = 2 + \frac{32}{x^3}$

For maximum or minimum values of S , consider

$$\frac{dS}{dx} = 0$$

$$\Rightarrow 2x = \frac{16}{x^2} \Rightarrow x^3 = 8 \Rightarrow x = 2 \text{ m}$$

$$\text{At } x = 2, \frac{d^2S}{dx^2} = 2 + \frac{32}{2^3} = 2 + 4 = 6 > 0$$

$\therefore S$ is minimum when $x = 2$

Now as $x^2 y = 4$, therefore $y = 1$

Thus, $x = 2y$

(iv) (b) : Since, surface area is minimum when $x = 2y$, therefore cost of material will be least when $x = 2y$.
 Thus, cost of material will be least when width is equal to twice of its depth.

(v) (c) : Since, minimum surface area
 $= x^2 + 4xy = 2^2 + 4 \times 2 \times 1 = 12 \text{ m}^2$ and
 cost per $\text{m}^2 = ₹ 360$

\therefore Minimum cost is $= ₹ (12 \times 360) = ₹ 4320$

19. The given curve is $y = x^3$.

$$\Rightarrow \frac{dy}{dx} = 3x^2$$

Slope of tangent at $(1, 1)$ is $\left(\frac{dy}{dx}\right)_{(1,1)} = 3(1)^2 = 3$

Equation of tangent at $(1, 1)$ is

$$y - 1 = 3(x - 1) \Rightarrow y - 1 = 3x - 3 \Rightarrow 3x - y = 2$$

Equation of normal at $(1, 1)$ is

$$y - 1 = \frac{-1}{3}(x - 1) \Rightarrow x + 3y = 4$$

20. We write, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$

$$= \tan^{-1}\left(\frac{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) + 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right) = \tan^{-1}\left(\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = \frac{\pi}{4} - \frac{x}{2}$$

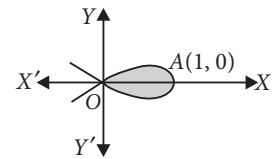
21. Given curve is $y^2 = x(1 - x)^2$

If $y = 0$, then $x(1 - x)^2 = 0 \Rightarrow x = 0, x = 1$

$$\therefore \text{Required area} = 2 \int_0^1 \sqrt{x(1-x)} dx$$

$$= 2 \left[\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} \right]_0^1$$

$$= 2 \left[\frac{2}{3}(1) - \left(\frac{2}{5}(1)\right) \right] = \frac{8}{15} \text{ sq. unit}$$



22. Let E_1, E_2 and A denote the events defined as follows :

E_1 = person selected is man

E_2 = person selected is woman

A = person selected is good orator

We have, $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$

Now, $P(A|E_1) = \frac{5}{100}$ and $P(A|E_2) = \frac{25}{1000}$

Required probability is

$$P(E_1|A) = \frac{P(E_1) \times P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{100}}{\frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{25}{1000}} = \frac{50}{75} = \frac{2}{3}$$

$$23. \text{ Let } I = \int \frac{1}{\sqrt{1-\sin x}} dx = \int \frac{1}{\sqrt{1+\cos\left(\frac{\pi}{2}+x\right)}} dx$$

$$= \int \frac{1}{\sqrt{2\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}} dx = \frac{1}{\sqrt{2}} \int \sec\left(\frac{\pi}{4}+\frac{x}{2}\right) dx$$

$$= \frac{2}{\sqrt{2}} \log \left| \tan\left(\frac{\pi}{4}+\frac{\pi}{8}+\frac{x}{4}\right) \right| + C$$

$$= \sqrt{2} \log \left| \tan\left(\frac{3\pi}{8}+\frac{x}{4}\right) \right| + C$$

OR

$$\text{Let } I = \int \frac{dx}{1-\sin 2x}$$

$$\text{Put } \tan x = t \Rightarrow dx = \frac{dt}{1+t^2} \text{ and } \sin 2x = \frac{2t}{1+t^2}$$

$$\therefore I = \int \frac{dt}{1+t^2-2t} = \int \frac{dt}{(t-1)^2} = \frac{-1}{t-1} + C$$

$$\Rightarrow I = \frac{-1}{\tan x - 1} + C$$

$$24. \text{ We have, } AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} \\ = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

$$\text{and } A \cdot (\text{adj} A) = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

$\therefore A \cdot (\text{adj} A) = AA^T$ is known, so equating the two expressions, we get

$$\begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$

We have, $10a + 3b = 13$ and $15a - 2b = 0$

On solving, we get $a = 2/5$ and $b = 3$

Thus, $5a + b = 2 + 3 = 5$

$$25. \text{ Let } \vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}, P \equiv (5, 6, -3) \text{ and } Q \equiv (3, 4, -2)$$

$$\therefore \overline{PQ} = (3-5)\hat{i} + (4-6)\hat{j} + (-2+3)\hat{k} = -2\hat{i} - 2\hat{j} + \hat{k}$$

Now the projection of \vec{a} on \overline{PQ}

$$= \frac{\vec{a} \cdot \overline{PQ}}{|\overline{PQ}|} = \frac{-4+6-6}{\sqrt{4+4+1}} = -\frac{4}{3}$$

OR

$$\text{We have, } \vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k} \text{ and } \vec{b} = 3\hat{i} + 2\hat{k}$$

$$\therefore 2\vec{a} = 8\hat{i} + 6\hat{j} + 4\hat{k}$$

$$\text{Now, } \vec{b} \times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 8 & 6 & 4 \end{vmatrix} = -12\hat{i} + 4\hat{j} + 18\hat{k}$$

$$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{(-12)^2 + 4^2 + (18)^2} = 22$$

26. Total no. of aces = 4

Also, X can take the values 0, 1, 2

$$\therefore P(X=0) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{188}{221},$$

$$P(X=1) = \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{32}{221} \text{ and } P(X=2) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{1}{221}$$

The probability distribution of X is as follows:

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

27. We have, $y = \sin^{-1} x$.

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{d}{dx} \left(\sqrt{1-x^2} \cdot \frac{dy}{dx} \right) = 0$$

$$\Rightarrow \sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx} \left(\sqrt{1-x^2} \right) = 0$$

$$\Rightarrow \sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} - \frac{dy}{dx} \cdot \frac{2x}{2\sqrt{1-x^2}} = 0$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$$

28. We have, $\frac{dy}{dx} = \frac{y(1+x)}{x(y-1)}$

$$\Rightarrow \left(\frac{y-1}{y} \right) dy = \left(\frac{1+x}{x} \right) dx$$

$$\Rightarrow \int \left(1 - \frac{1}{y} \right) dy = \int \left(\frac{1}{x} + 1 \right) dx + C_1$$

$$\Rightarrow y - \log |y| = \log |x| + x + C_1 \Rightarrow x - y + \log |xy| = C, \text{ where } C = -C_1$$

OR

$$\text{Given, } \log \left(\frac{dy}{dx} \right) = 3x + 4y \Rightarrow \frac{dy}{dx} = e^{3x+4y}$$

$$\Rightarrow \frac{dy}{dx} = e^{3x} e^{4y} \Rightarrow \int e^{-4y} dy = \int e^{3x} dx$$

$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

$$\text{At } x=0, y=0, \text{ then } \frac{-1}{4} = \frac{1}{3} + C \Rightarrow C = \frac{-7}{12}$$

$\therefore \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow 4e^{3x} + 3e^{-4y} - 7 = 0$, which is the required solution.

29. Let $P(x, y)$ be an arbitrary point on the given curve. The equation of the normal to the given curve at (x, y) is $Y - y = -\frac{1}{\frac{dy}{dx}}(X - x)$

It is given that the normal at every point passes through a fixed point (α, β) (say).

Therefore, $\beta - y = -\frac{dx}{dy}(\alpha - x)$

$$\Rightarrow (x - \alpha)dx + (y - \beta)dy = 0$$

Integrating both sides, we get

$$\int (x - \alpha)dx + \int (y - \beta)dy = C$$

$$\Rightarrow \frac{(x - \alpha)^2}{2} + \frac{(y - \beta)^2}{2} = C$$

$$\Rightarrow (x - \alpha)^2 + (y - \beta)^2 = r^2, \text{ where } r^2 = 2C$$

Clearly, this equation represents a circle, having centre at (α, β) and radius r .

30. Let $A(x, y)$ be the required point which is closest to the point $B(1, 4)$. Then, the distance AB should be minimum and therefore AB^2 should be minimum.

$$\text{Now, } AB^2 = (x - 1)^2 + (y - 4)^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y - 4)^2$$

$$= \frac{(y^4 - 32y + 68)}{4}$$

$$\text{Let } f(y) = \frac{y^4 - 32y + 68}{4}$$

$$\text{Then, } f'(y) = \frac{4y^3 - 32}{4} = y^3 - 8 \text{ and } f''(y) = 3y^2$$

$$\text{Now, } f'(y) = 0 \Rightarrow y^3 - 8 = 0 \Rightarrow y = 2$$

$$\text{Also, } f''(2) = 3 \times 4 = 12 > 0$$

So, $y = 2$ is a point of minima.

$$\text{Now, } y = 2 \Rightarrow x = \frac{y^2}{2} = \frac{4}{2} = 2.$$

So, the required point is $(2, 2)$.

31. We have,

$$y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$$

$$ax^2 + bx(x-a) + c(x-a)(x-b)$$

$$\Rightarrow y = \frac{+ (x-a)(x-b)(x-c)}{(x-a)(x-b)(x-c)}$$

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$$\Rightarrow y = \frac{x^3}{(x-a)(x-b)(x-c)}$$

$$\Rightarrow \log y = \log \left\{ \frac{x^3}{(x-a)(x-b)(x-c)} \right\}$$

$$\Rightarrow \log y = 3 \log x - \{\log(x-a) + \log(x-b) + \log(x-c)\}$$

On differentiating w.r.t. x , we get

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} - \left\{ \frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} \right\}$$

$$\Rightarrow \frac{dy}{dx} = y \left\{ \left(\frac{1}{x} - \frac{1}{x-a} \right) + \left(\frac{1}{x} - \frac{1}{x-b} \right) + \left(\frac{1}{x} - \frac{1}{x-c} \right) \right\}$$

$$\Rightarrow \frac{dy}{dx} = y \left\{ \frac{-a}{x(x-a)} + \frac{(-b)}{x(x-b)} + \frac{(-c)}{x(x-c)} \right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} \left\{ \frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x} \right\}$$

OR

$$\text{Let } y = \frac{2^{x^2+x+1} \cdot \sqrt{4x-1}}{(x^2-1)^{3/2}}$$

$$\Rightarrow \log y = \log \left[\frac{2^{x^2+x+1} \cdot \sqrt{4x-1}}{(x^2-1)^{3/2}} \right]$$

$$= (x^2 + x + 1) \log 2 + \frac{1}{2} \log(4x-1) - \frac{3}{2} \log(x^2-1)$$

Differentiating both sides w.r.t. x , we get

$$\frac{1}{y} \frac{dy}{dx} = \log 2 \cdot \frac{d}{dx}(x^2 + x + 1) + \frac{1}{2} \cdot \frac{1}{4x-1} \times \frac{d}{dx}(4x-1)$$

$$- \frac{3}{2} \cdot \frac{1}{x^2-1} \cdot \frac{d}{dx}(x^2-1)$$

$$= (2x+1) \log 2 + \frac{1}{2(4x-1)} \cdot 4 - \frac{3}{2(x^2-1)} \cdot 2x$$

$$\Rightarrow \frac{dy}{dx} = y \left[(2x+1) \log 2 + \frac{2}{4x-1} - \frac{3x}{x^2-1} \right]$$

$$32. \text{ Let } y = f(x) = \frac{2x+3}{x-3} \quad \dots(i)$$

Let $x_1, x_2 \in A = R - \{3\}$ such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{2x_1+3}{x_1-3} = \frac{2x_2+3}{x_2-3}$$

$$\Rightarrow (2x_1+3)(x_2-3) = (2x_2+3)(x_1-3)$$

$$\Rightarrow 2x_1x_2 - 6x_1 + 3x_2 - 9 = 2x_1x_2 - 6x_2 + 3x_1 - 9$$

$$\Rightarrow -6x_1 + 3x_2 = -6x_2 + 3x_1$$

$$\Rightarrow 9x_1 = 9x_2 \Rightarrow x_1 = x_2$$

$$\text{Now, } f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

So $f(x)$ is one-one.

$$\text{For onto, let } y = \frac{2x+3}{x-3} \Rightarrow xy - 3y = 2x + 3$$

$$\Rightarrow xy - 2x = 3y + 3 \Rightarrow x(y - 2) = 3(y + 1)$$

$$\Rightarrow x = \frac{3(y+1)}{(y-2)} \quad \dots(ii)$$

Equation (ii) is defined for all real values of y except 2 which is same as given set $B = R - \{2\}$.

Thus, for every $y \in B$, there exist $x = \frac{3(y+1)}{y-2} \in A$ such that $f(x) = y$

Hence, function f is onto.

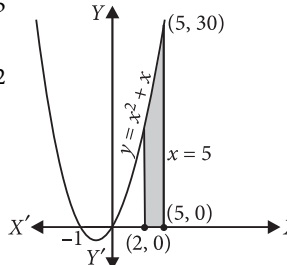
33. Given curve is $y = x^2 + x$.

Clearly, required area = Area of shaded region

$$= \int_2^5 (x^2 + x) dx = \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_2^5$$

$$= \left[\frac{125}{3} + \frac{25}{2} - \left(\frac{8}{3} + 2 \right) \right]$$

$$= \left[\frac{250 + 75}{6} - \frac{14}{3} \right]$$

$$= \frac{325}{6} - \frac{14}{3} = \frac{325 - 28}{6} = \frac{297}{6} \text{ sq. units}$$


34. Let $I = \int_0^1 \tan^{-1} x dx = \int_0^1 \tan^{-1} x \cdot 1 dx$

$$= [\tan^{-1} x \cdot x]_0^1 - \int_0^1 \frac{1}{1+x^2} \cdot x dx$$

$$= \left(\frac{\pi}{4} - 0 \right) - \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} dx = \frac{\pi}{4} - \frac{1}{2} I_1 \quad \dots(i)$$

Consider $I_1 = \int_0^1 \frac{2x}{1+x^2} dx$

Put $1 + x^2 = t \Rightarrow 2x dx = dt$

When $x = 0, t = 1$ and when $x = 1, t = 2$

$$\therefore I_1 = \int_1^2 \frac{1}{t} dt = [\log t]_1^2 = \log 2 - \log 1 = \log 2 \quad \dots(ii)$$

$$\Rightarrow I = \frac{\pi}{4} - \log \sqrt{2} \quad \text{[From (i) and (ii)]}$$

OR

Let $I = \int_1^2 \frac{dx}{x(1+x^2)}$

Consider, $\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}$

$$\Rightarrow 1 = A(1+x^2) + (Bx+C) \cdot x$$

$$\Rightarrow 1 = x^2(A+B) + Cx + A$$

On equating the coefficient of x^2, x and the constant term from both sides, we get $A = 1, B = -1$ and $C = 0$

$$\therefore I = \int_1^2 \frac{1}{x} dx + \int_1^2 \frac{-x}{1+x^2} dx$$

$$= [\log x]_1^2 - \int_1^2 \frac{x}{1+x^2} dx = \log 2 - \int_1^2 \frac{x dx}{1+x^2}$$

Put $1 + x^2 = t \Rightarrow 2x dx = dt$

When $x = 1, t = 2$ and when $x = 2, t = 5$

$$\therefore I = \log 2 - \frac{1}{2} \int_2^5 \frac{1}{t} dt = \log 2 - \frac{1}{2} [\log t]_2^5$$

$$= \log 2 - \frac{1}{2} [\log 5 - \log 2]$$

$$= \log 2 - \frac{1}{2} \log \left(\frac{5}{2} \right) = \frac{1}{2} \log \left(\frac{8}{5} \right)$$

35. Since, $f(x)$ is continuous at $x = 0$, therefore

$$f(0) = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) \quad \dots(i)$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} \left(\frac{4(1-\sqrt{1-x})}{x} \right)$$

$$= 4 \lim_{x \rightarrow 0} \left(\frac{1-(1-x)}{x(1+\sqrt{1-x})} \right) = 4 \lim_{x \rightarrow 0} \left(\frac{1-1+x}{x(1+\sqrt{1-x})} \right)$$

$$= 4 \left(\frac{1}{(1+1)} \right) = \frac{4}{2} = 2$$

From (i), we get $f(0) = 2$.

36. Here we need to find, the equation of the line through the point $(1, 1, -3)$ and perpendicular to the lines

$$\frac{x-2}{2} = \frac{y+3}{1} = \frac{z-0}{-3} \quad \dots(i)$$

and $\frac{x-3}{1} = \frac{y+5}{1} = \frac{z-0}{1} \quad \dots(ii)$

Let the direction ratios of required line are a, b, c . Then, equations of this line is given by

$$\frac{x-1}{a} = \frac{y-1}{b} = \frac{z+3}{c} \quad \dots(iii)$$

Direction ratios of line (i) are 2, 1, -3 and line is perpendicular to line (iii) having direction ratios a, b, c

$$\therefore 2a + b - 3c = 0 \quad \dots(iv)$$

Similarly $a + b + c = 0 \quad \dots(v)$

Solving equation (iv) and (v), we get

$$\frac{a}{4} = \frac{-b}{5} = \frac{c}{1}$$

\therefore From equation (iii), required equation of the line

$$\text{is } \frac{x-1}{4} = \frac{y-1}{-5} = \frac{z+3}{1}$$

Its vector equation is $\vec{r} = (\hat{i} + \hat{j} - 3\hat{k}) + \lambda(4\hat{i} - 5\hat{j} + \hat{k})$

OR

The equation of the plane passing through the point $A(3, 2, -5)$ is given by

$$a(x-3) + b(y-2) + c(z+5) = 0 \quad \dots(i)$$

If it passes through $B(-1, 4, -3)$ and $C(-3, 8, -5)$,

we get

$$a(-1 - 3) + b(4 - 2) + c(-3 + 5) = 0$$

$$\Rightarrow -4a + 2b + 2c = 0$$

$$\Rightarrow 2a - b - c = 0 \quad \dots(\text{ii})$$

$$\text{Also } a(-3 - 3) + b(8 - 2) + c(-5 + 5) = 0$$

$$\Rightarrow -6a + 6b + 0c = 0$$

$$\Rightarrow a - b - 0c = 0 \quad \dots(\text{iii})$$

Solving (ii) and (iii) by cross multiplication method,

we get

$$\frac{a}{(0-1)} = \frac{b}{(-1-0)} = \frac{c}{(-2+1)}$$

$$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1} = k \text{ (say)} \Rightarrow a = k, b = k, c = k.$$

Putting $a = k, b = k$ and $c = k$ in (i), we get

$$(x - 3) + (y - 2) + (z + 5) = 0$$

$$\Rightarrow x + y + z = 0.$$

Thus, the equation of the plane passing through the points $A(3, 2, -5), B(-1, 4, -3)$ and $C(-3, 8, -5)$ is

$$x + y + z = 0.$$

Clearly, the fourth point $D(-3, 2, 1)$ also satisfies $x + y + z = 0$.

Hence, equation of the plane containing the given points is $x + y + z = 0$.

37. Converting inequations into equations, we get

$$x + y = 60 \quad \dots(\text{i})$$

$$x = 40 \quad \dots(\text{ii})$$

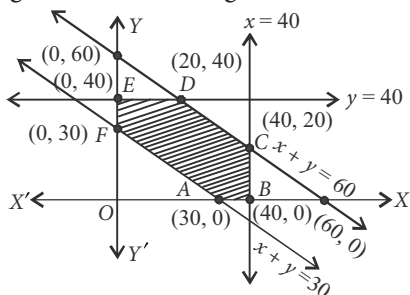
$$y = 40 \quad \dots(\text{iii})$$

$$x + y = 30 \quad \dots(\text{iv})$$

$$x = 0 \quad \dots(\text{v})$$

$$\text{and } y = 0 \quad \dots(\text{vi})$$

Let us draw the graph of equations (i) to (vi). The feasible region is shown in figure.



The coordinates of the corner points of the feasible region are $A(30, 0), B(40, 0), C(40, 20), D(20, 40), E(0, 40)$ and $F(0, 30)$.

Let us evaluate Z at these points.

Corner points	Value of $Z = 3x + 4y + 270$
$A(30, 0)$	$360 \leftarrow$ Minimum
$B(40, 0)$	390
$C(40, 20)$	470

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$D(20, 40)$	490
$E(0, 40)$	430
$F(0, 30)$	390

From the table, the minimum value of Z is 360, which is attained at the point $A(30, 0)$.

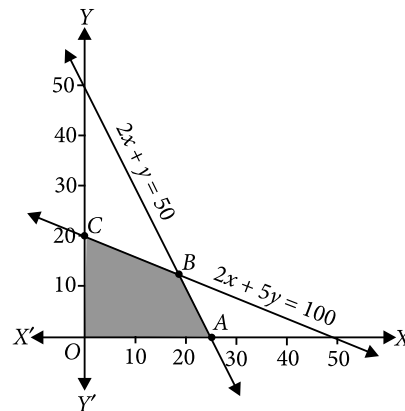
OR

Converting inequations into equations, we get

$$2x + 5y = 100, 2x + y = 50, x = 0 \text{ and } y = 0$$

$$\text{i.e., } \frac{x}{50} + \frac{y}{20} = 1, \frac{x}{25} + \frac{y}{50} = 1, x = 0 \text{ and } y = 0$$

Let us draw the graph of above equations.



Clearly, the feasible region is $OABCO$, which is shaded in the figure.

Here, B is the point of intersection of lines $2x + 5y = 100$

$$\text{and } 2x + y = 50 \text{ i.e., } B = \left(\frac{75}{4}, \frac{25}{2} \right)$$

We have corner points $A(25, 0), B\left(\frac{75}{4}, \frac{25}{2}\right)$ and $C(0, 20)$.

The values of the objective function $Z = x + y$ at these points are

$$Z(A) = 25 + 0 = 25$$

$$Z(B) = \frac{75}{4} + \frac{25}{2} = 31.25$$

$$Z(C) = 0 + 20 = 20$$

$$Z(O) = 0 + 0 = 0$$

The maximum value of Z is 31.25, which is attained at $\left(\frac{75}{4}, \frac{25}{2}\right)$.

$$38. \text{ We have, } A = \begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= 2(-4 + 1) - 3(6 + 1) + 7(3 + 2) = -6 - 21 + 35$$

$$= 8 \neq 0. \text{ So } A^{-1} \text{ exist.}$$

The cofactors of elements of A are

$$C_{11} = -3, C_{21} = 1, C_{31} = 11$$

$$C_{12} = -7, C_{22} = -3, C_{32} = 23$$

$$C_{13} = 5, C_{23} = 1, C_{33} = -13$$

$$\therefore \text{adj} A = \begin{pmatrix} -3 & 1 & 11 \\ -7 & -3 & 23 \\ 5 & 1 & -13 \end{pmatrix}$$

$$\text{and } A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{8} \begin{pmatrix} -3 & 1 & 11 \\ -7 & -3 & 23 \\ 5 & 1 & -13 \end{pmatrix}$$

Given system of equations is

$$2x + 3y + 7z = 12$$

$$3x - 2y - z = 0$$

$$x + y + 2z = 4$$

which can be written in matrix form as

$$\begin{pmatrix} 2 & 3 & 7 \\ 3 & -2 & -1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 4 \end{pmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B$$

$$\Rightarrow X = \frac{1}{8} \begin{pmatrix} -3 & 1 & 11 \\ -7 & -3 & 23 \\ 5 & 1 & -13 \end{pmatrix} \begin{pmatrix} 12 \\ 0 \\ 4 \end{pmatrix}$$

$$\text{Now, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = X = \frac{1}{8} \begin{pmatrix} -36+0+44 \\ -84-0+92 \\ 60+0-52 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Comparing we get $x = 1, y = 1, z = 1$

OR

Here,

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

We know, if A is non-singular matrix,

then $(\text{adj} A^{-1}) = (\text{adj} A)^{-1}$, so we will find $(\text{adj} A)^{-1}$.

The cofactors of elements of A are

$$A_{11} = 14 \quad A_{12} = 11 \quad A_{13} = -5$$

$$A_{21} = 11 \quad A_{22} = 4 \quad A_{23} = -3$$

$$A_{31} = -5 \quad A_{32} = -3 \quad A_{33} = -1$$

$$\therefore B = \text{adj} A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\therefore |B| = |\text{adj} A| = 14(-4-9) - 11(-11-15) - 5(-33+20)$$

$$= -182 + 286 + 65 = 169 \neq 0$$

Cofactors of B are

$$B_{11} = -13 \quad B_{12} = 26 \quad B_{13} = -13$$

$$B_{21} = 26 \quad B_{22} = -39 \quad B_{23} = -13$$

$$B_{31} = -13 \quad B_{32} = -13 \quad B_{33} = -65$$

$$\therefore \text{adj} B = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix} = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$\text{and } B^{-1} = (\text{adj} B)^{-1} = \frac{1}{169} \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

